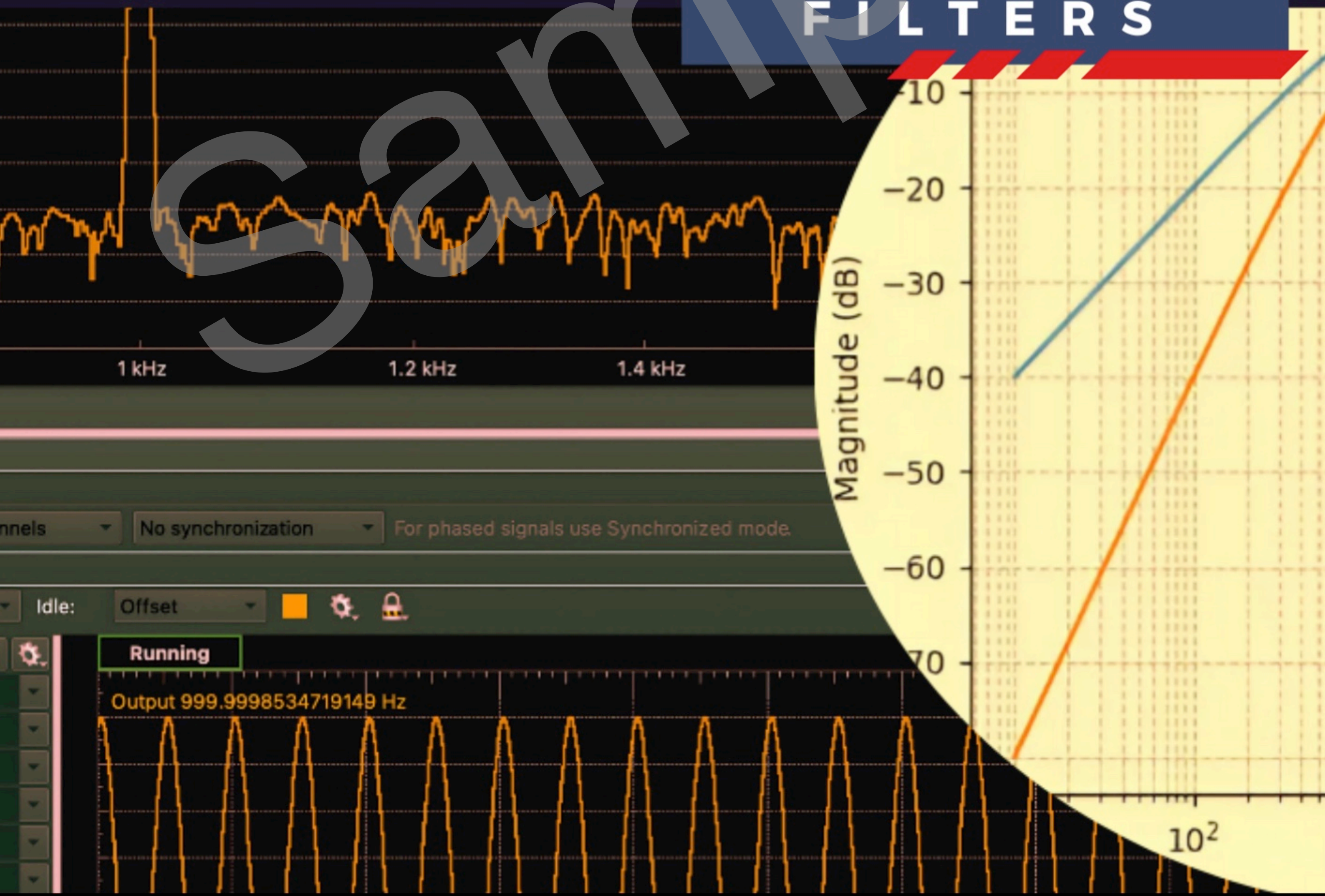
1ST EDITION

INTRODUCTION TO ELECTRONICS

AN INTRODUCTION TO THE FOUNDATIONAL CONCEPTS OF RC AND RL FILTERS, EMPOWERING YOU TO UNDERSTAND, DESIGN, AND ANALYZE THESE ESSENTIAL CIRCUITS.

DR PETER DALMARIS

1ST ORDER FILTERS



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Preface

The idea for this book emerged from a simple observation: students often understand basic electronic components well but struggle when asked to apply that knowledge to real-world signal control problems. Filters, though conceptually straightforward, frequently appear abstract or unnecessarily complicated when introduced solely through mathematical formalisms. Yet, filters are everywhere, quietly performing critical functions in audio equipment, communication devices, industrial sensors, and embedded systems.

I wrote this book to address this gap. My goal is to teach first-order filters in a way that is both rigorous and approachable, blending theory with hands-on practice.

Rather than treating first-order filters as a brief stepping stone towards more complex subjects, this book treats them as powerful tools that deserve careful attention in their own right.

Throughout this book, you will find a continuous emphasis on practice. You will use Python scripts to simulate circuits, online simulators to quickly prototype designs, and real-world breadboard experiments to observe circuit behaviour with oscilloscopes and spectrum analysers. This combination of simulation and physical experimentation is intended to make the material tangible and memorable. It is through building, observing, and analysing that theory becomes truly internalised.

This book assumes that you are already familiar with the fundamentals of electronics, particularly the behaviour of resistors, capacitors, and inductors, as well as basic circuit analysis techniques. If you have completed the Introduction to Electronics course or possess similar knowledge, you are fully prepared to embark on the journey ahead.

Learning about filters is not only about gaining technical skills. It is also about learning to think critically about signals, noise, and system behaviour. It is about recognising the subtle but vital roles that small circuits play in the reliability and performance of much larger systems. My hope is that, by the end of this book, you will not only know how to design and analyse first-order filters, but also appreciate their elegance and importance in the broader context of electronics.

I encourage you to approach the material with curiosity and patience. Build the circuits, run the simulations, question the results, and take the time to explore the behaviour of filters beyond the examples provided. The deeper your engagement with the material, the stronger and more intuitive your understanding will become.

Welcome to the study of first-order filters. I am excited to be your guide on this practical and rewarding journey.

How to Use This Book

This book is designed to be both a study guide and a practical reference. To gain the maximum benefit from the material, it is important to approach it as an active participant rather than a passive reader.

Each chapter builds systematically on the knowledge and skills introduced earlier. Concepts are introduced progressively, beginning with a general understanding of filters and their classifications, and moving towards detailed analyses and practical experiments with first-order filter circuits. It is recommended that you read the chapters sequentially, as each topic prepares the foundation for the next.

Wherever simulations are presented, you are encouraged to run the provided Python scripts on your own system. These simulations allow you to visualise filter behaviours that are difficult to grasp from static text alone. The scripts are designed to be clear and accessible, even for readers with only basic Python programming experience. Modifying the parameters and observing how the results change is an excellent way to deepen your understanding.

You will find all Python scripts on Github at https://github.com/futureshocked/introduction_to_electronics_filters

In addition to Python simulations, many examples reference online circuit simulators. These simulators allow you to experiment with filter circuits interactively. It is highly recommended that you reproduce these circuits yourself, adjust component values, and observe how these changes affect circuit behaviour. This hands-on exploration will reinforce the theoretical principles discussed in the text.

Real-world experiments form another vital component of the learning experience. When possible, you should construct the simple filter circuits described in the book on a breadboard. Using basic laboratory instruments such as oscilloscopes and spectrum analysers, you will be able to observe signal behaviour directly. These experiments are chosen to be accessible with modest equipment, yet they provide invaluable experience in measurement and analysis.

As you work through the material, it is important to keep a notebook or digital journal of your observations, measurements, and conclusions. Reflecting on what you observe, especially when simulation results differ from physical measurements, will build a deeper and more critical understanding of electronic behaviour.

Finally, although this book focuses specifically on first-order filters, you will occasionally encounter references to second-order filters and more advanced topics. These references are included to provide context and to prepare you for future learning. However, it is not necessary to understand higher-order filters in detail before mastering the first-order designs presented here.

Your success with this book depends less on speed and more on deliberate practice. Take the time to build, test, and truly understand each circuit. By doing so, you will develop both the theoretical knowledge and the practical confidence that define an effective electronics practitioner.

Did you find an error?

If you have found an error, please let me know.

Go to *https://connect.techexplorations.com/bugs/introduction-to-electronics*, and fill in the form.

I’ll get it fixed and update the book.

About the author

Dr. Peter Dalmaris is an educator, an electrical engineer, an electronics hobbyist, and a Maker. Creator of online video courses on DIY electronics and author of several technical books. Peter has recently released his book 'Maker Education Revolution', a book about how Making is changing the way we learn and teach in the 21st century.

As a Chief Tech Explorer since 2013 at Tech Explorations, the company he founded in Sydney, Australia, Peter's mission is to explore technology and help educate the world.

Tech Explorations offers educational courses and Bootcamps for electronics hobbyists, STEM students, and STEM teachers.

A lifelong learner, Peter's core skill lies in explaining difficult concepts through video and text. With over 15 years of tertiary teaching experience, Peter has developed a simple yet comprehensive style of teaching that students from all around the world appreciate.

His passion for technology and the world of DIY open-source hardware has been a dominant driver that has guided his personal development and his work through Tech Explorations.

About Tech Explorations

Tech Explorations creates educational products for students and hobbyists of electronics who would rather utilise their time making awesome gadgets instead of searching endlessly through blog posts and YouTube videos. We deliver high-quality instructional videos and books through our online learning platform, techexplorations.com. Our priority is supporting our students through their learning journey We do this through our dedicated online community and course forums.

Founded in 2013 by Peter Dalmaris, Tech Explorations was created after Peter realised how difficult it was to find high-quality definitive guides for the Arduino, written or produced by creators who responded to their reader questions. Peter was frustrated having to search for YouTube videos and blog articles that rarely seemed to be made to convey knowledge.

He decided to create Teach Explorations to produce the educational content he wished he could find back then. Tech Explorations courses are designed to be comprehensive, definitive, and practical. Whether delivered via video, ebook, blog, or email, our delivery is personal and conversational. It is like having a friend showing you something neat... the "AHA" moments flow!

Peter left his career in Academia after his passion for electronics and making was rekindled with the arrival of his first Arduino. Although he was an electronics hobbyist from a young age, something that led him to study electrical and electronics engineering at University, the Arduino signaled a revolution in how electronics is taught and learned. Peter decided to be a part of this revolution and has never looked back.

We know that even today, with all the world’s information at your fingertips, thanks to Google and all the components of the world one click away, thanks to eBay, the life of the electronics hobbyist is not easy. Busy lifestyles leave little time for your hobby, and you want this time to count.

We want to help you enjoy your hobby and learn amazing practical things that you can use to make your own awesome gadgets. Electronics is a rewarding hobby. Science, engineering, mathematics, art, and curiosity converge in a tiny circuit with several components. We want to help you take this journey without delays and frustrations.

Over 150,000 people across the world have used our courses. From prototyping electronics with the Arduino prototyping board to learning full-stack development with the Raspberry Pi or designing professional-looking printed circuit boards for their awesome gadgets, our students enjoyed taking our courses and improved their making skills dramatically.

Here's what some of them had to say:

> *"I'm about half way through this course and I am learning so much. Peter is an outstanding instructor. I recommend this course if you really want to learn about the versatility of the amazing Raspberry Pi" -- Scott*

> *"The objectives of this course are uniquely defined and very useful. The instructor explains the material very clearly." -- Huan*

> *"Logical for the beginner. Many things that I did not know so far about Arduino but easy to understand. Also the voice is easy to understand which is unlike many courses about microcontrollers that I have*

STARTED in the past. Thanks" -- Anthony

Please check out our courses at techexplorations.com and let us be part of your tech adventures.

Back cover text

Introduction to Electronics: Filters is your comprehensive guide to understanding, designing, and applying first-order electronic filters using resistors, capacitors, and inductors. Whether you are a student, maker, or educator, this book demystifies the theory behind RC and RL filters and bridges the gap between concepts and real-world applications through simulation and experimentation.

From the basics of frequency response and phase shift to hands-on breadboard builds and Python-based simulations, this book offers a deeply practical learning experience. You will learn to analyse filters using Bode plots and phasors, and explore applications in audio tone shaping, sensor signal conditioning, noise reduction, and power supply filtering. As you progress, you'll build, measure, simulate, and tune filters using modern tools like CircuitLab, Python, and the Analog Discovery 3.

Each chapter includes thoughtfully crafted activities that reinforce learning by doing—designing filters for specific tasks, simulating dynamic behaviour, and observing how theory translates into performance.

Inside you'll find:

- A clear introduction to the fundamentals of electronic filters
- Detailed explanations of RC and RL filters, cutoff frequency, and phase
- Guided activities using both simulation and hardware tools

- Real-life applications in audio, sensors, power supplies, and more
- A beginner-friendly primer on Python and algebra for electronics

Whether you're working through simulations or experimenting with real components on your workbench, this book will help you develop a solid understanding of electronic filters and their role in practical circuits.

About the author

Peter Dalmaris is an educator, engineer, and lifelong learner. He is the founder of Tech Explorations, where he creates online courses and books that empower electronics enthusiasts, STEM educators, and students around the world. With a background in engineering and academic teaching, Peter is known for his hands-on, accessible approach to technical education. His work combines theory with practice, helping learners of all levels build confidence through making and experimentation.

0 Contents

Part I

Introduction

1 Course Overview and Objectives

This book offers a practical and structured exploration of first-order filters, intended for readers who have completed the Introduction to Electronics course or possess equivalent foundational knowledge. It is designed for students, hobbyists, and professionals who wish to build a working understanding of filter circuits, both in theory and in practice.

The primary focus of this book is to bridge theoretical knowledge with practical skills. Readers will not only learn the fundamental concepts underlying first-order filters, but also how to apply these concepts through circuit simulations and hands-on experiments. The approach taken throughout the text emphasises understanding through doing. Theory is introduced progressively and supported by meaningful applications.

One of the core methods employed in this book involves the use of Python scripts. These scripts are provided to simulate various filter behaviours, allowing the reader to visualise the effect of circuit parameters on the performance of first-order filters. Python, with its powerful numerical and graphical libraries, offers an accessible and flexible environment for rapid experimentation and deep analysis. The simulations presented throughout the chapters are simple enough to run on any modern computer yet sophisticated enough to yield meaningful insights.

Alongside Python-based simulations, the book makes extensive use of online circuit simulators. These web-based tools allow readers to design, modify, and test filter circuits without the need for physical components. Using an online simulator, it is possible to quickly iterate through different configurations, examine time-domain and frequency-domain behaviours, and understand the impact of design decisions. Online simulators provide an invaluable platform for bridging the gap between theoretical design and practical implementation.

While simulation is a powerful educational tool, nothing replaces the tactile experience of building real circuits. For this reason, this book also guides readers through practical experiments on a breadboard. These experiments involve assembling basic first-order filter circuits using resistors, capacitors, and inductors, and observing their behaviour using laboratory instruments such as oscilloscopes and spectrum analysers. By directly measuring input and output signals, readers will gain an intuitive and technical understanding of key filter characteristics, such as cutoff frequency, phase shift, and roll-off.

Throughout the book, emphasis is placed on the connection between simulation and measurement. Readers are encouraged to compare simulated results with real-world measurements, to analyse any discrepancies, and to understand the reasons behind them. This process of continuous verification not only reinforces learning but also builds critical engineering habits.

The study of first-order filters in this book is not presented as a mere preliminary step towards higher-order filters. Instead, first-order filters are treated as valuable and powerful tools in their own right. Many practical applications rely solely on first-order designs due to their simplicity, reliability, and efficiency. Readers will encounter numerous examples drawn from real-world electronics, including audio processing, sensor interfacing, power management, and communication systems.

By the end of the book, readers will have acquired a robust understanding of first-order filters. They will be able to design, simulate, build, and measure the performance of basic filter circuits with confidence. They will also be well-prepared for more advanced studies in second-order filters, active filter design, and digital signal processing, should they choose to pursue them.

This book is structured in a progressive manner, beginning with an introduction to filters and their applications, proceeding to an in-depth exploration of first-order low-pass, high-pass, band-pass, and band-stop filters, and concluding with practical projects and case studies. Each chapter builds systematically on the previous one, ensuring that knowledge and skills are developed logically and thoroughly.

The objective of this book is to make the study of first-order filters accessible, practical, and engaging. Through a combination of theory, simulation, and hands-on experimentation, readers will achieve a deep and enduring understanding of this essential topic in electronics.

2 Review of software and hardware tools used

This book is designed to be practical and accessible. To gain the full educational benefit, you will need access to a small set of software tools, and optionally, basic hardware tools.

While building physical circuits can greatly enhance your understanding, it is entirely possible to complete all simulations, exercises, and projects in this book using only Python and a circuit simulation platform.

In this chapter, we review the software and hardware tools recommended for following the activities described throughout the book.

2.1 Software tools

Software tools are central to the learning approach used in this book. Simulations allow you to visualise the behaviour of filters, to explore how component values affect circuit performance, and to verify theoretical predictions with practical examples. **Python environment** Python is used throughout the book to simulate the behaviour of first-order filters and to generate Visualisations such as time-domain plots and frequency responses.

The following software components are required:

- **Python 3.x**: A current version of Python is recommended (version 3.8 or newer).
- **NumPy**: A library for numerical operations and array handling.
- **SciPy**: A library that provides scientific computing tools, including signal processing functions.
- **Matplotlib**: A plotting library used to visualise signals and filter responses. Python can be installed directly from the official Python website, or via distributions such as Anaconda, which include all required libraries pre-installed. If necessary, libraries can be installed individually using `pip`.

All Python scripts presented in this book are provided with clear explanations and can be modified easily to explore different circuit parameters and signal conditions.

Online circuit simulators An online circuit simulator is the second essential tool. It enables you to construct and test filter circuits graphically, without requiring any physical components.

Recommended simulators include:

- **CircuitLab:** (http://circuitlab.com/):
  Provides an intuitive, real-time Visualisation of voltage and current behaviours.
- **Tinkercad Circuits** (https://www.tinkercad.com/circuits):

A comprehensive web platform that supports circuit simulation as well as Arduino integration, suitable for readers interested in embedded applications.

Using Python and an online circuit simulator, you can complete every simulation, exploration, and design exercise described in this book.

2.2 Hardware Tools (Optional)

While not required, having access to basic laboratory equipment can deepen your understanding of filter behaviour by allowing you to build real circuits and measure their performance directly. Readers who have access to these tools are encouraged to use them, but they are not necessary to complete the learning objectives of this book. The following hardware tools are considered optional:

Breadboard and components

A standard breadboard along with jumper wires, resistors (ranging from 100 Ω to $100k\Omega$), capacitors (from 10 nF to $10\,\mu$F), and inductors (from $10\,\mu$H to 10 mH) would allow you to physically build the filters studied in the book.

Analog Discovery 3 Analysis Tool I use the Analog Discovery 3 analysis tool, which combines oscilloscope, spectrum analyser, signal generator, and power supply capabilities in one compact device. You can use this or any similar multifunction equipment to perform the hands-on experiments described.

Signal Generator

A basic signal generator can provide the input signals required to test your filter circuits. In the absence of a dedicated signal generator, software applications for smartphones or computers can serve as an alternative.

Oscilloscope

An oscilloscope enables the measurement of the time-domain behaviour of input and output signals.

A simple two-channel digital oscilloscope with a bandwidth of at least 20 MHz is sufficient for the types of filters and signals discussed.

Spectrum analyser

A spectrum analyser, or an oscilloscope with an FFT mode, can be used to observe how filters attenuate or pass different frequency components.

This tool provides a frequency-domain view, complementing time-domain observations.

Although valuable for hands-on understanding, all experiments and filter characterisations in this book can be conducted fully using simulation environments.

The presence of physical hardware is therefore optional and not a barrier to successful completion of the material.

2.3 Table of required and optional tools

Category	Tools and Resources	Requirement
Software (Required)	Python 3.x, NumPy, SciPy, Matplotlib, Online simulators (Falstad, Tinkercad)	Required
Construction (Optional)	Breadboard, jumper wires, resistors, capacitors, inductors	Optional
Signal Generation (Optional)	Signal generator (hardware or smartphone app)	Optional
Measurement (Optional)	Oscilloscope, Spectrum analyser or FFT-enabled oscilloscope	Optional

Throughout this book, all essential experiments and observations are demonstrated using Python simulations and online circuit simulators. These platforms allow you to build a complete understanding of first-order filter theory and practice without the need for additional equipment. However, if you have access to the recommended optional tools, you are encouraged to replicate experiments physically.

Hands-on construction and measurement deepen intuition and expose you to real-world phenomena such as component tolerances and parasitic effects, which cannot be fully captured in simulations. Regardless of your available tools, this book is structured to ensure that you can achieve all of its learning objectives and complete all exercises using freely available software resources.

Part II

What is a filter?

3 Introduction to filters

Filters are essential building blocks in the design and analysis of electronic circuits. They allow engineers and hobbyists to control which signals pass through a system and which signals are attenuated or blocked. At their core, filters shape the flow of electrical energy, enabling the separation, enhancement, or suppression of different parts of a signal.

Filters are used across countless applications. In audio systems, filters can remove unwanted noise or adjust the tonal quality of sound. In communication systems, they are vital for selecting specific frequency bands. In power electronics, filters suppress voltage spikes and reduce electromagnetic interference. Filters are critical components of virtually all non-trivial electronic systems, and this is why a solid understanding of their behaviour is so important.

3.1 Filter applications

To give you a sense of the wide application of filters in all sorts of gadgets, I have constructed the table below (see next page).

3.2 Classification of Filters

In the broadest sense, filters can be classified into two main categories:

- **Analog filters** operate directly on continuous-time signals using resistors, capacitors, inductors, and active components like operational amplifiers.
- **Digital filters** work on discrete-time signals, manipulating digitized data using algorithms implemented in software or digital circuits.

Further classification is based on frequency behaviour:

- **Low-pass filters** allow low-frequency signals to pass while attenuating high-frequency signals.
- **High-pass filters** allow high-frequency signals to pass while attenuating low-frequency signals.

Application Area	Filter Purpose	Example
Audio Systems	Tone control, noise reduction	Arduino-based audio equaliser; microphone noise filtering
Communication Systems	Channel selection, signal separation	Arduino RF module (e.g., LoRa, NRF24L01) signal filtering
Power Electronics	Noise suppression, voltage smoothing	Arduino voltage regulator output filter (buck converter smoothing)
Medical Devices	Signal conditioning for sensors	Arduino-based heart rate monitor (filtering noisy sensor data)
Instrumentation Systems	Measurement accuracy improvement	Arduino oscilloscope shield (input signal conditioning)
Motor Control	Reducing electrical noise from motors	Arduino motor driver circuits (filtering PWM signals)
Sensor Interfaces	Smoothing noisy analog sensor readings	Arduino analog input filters (temperature sensor, light sensor)
Wireless Systems	Reducing interference in wireless communication	Arduino Wi-Fi or Bluetooth projects (filtering power supply noise)
Data Acquisition Systems	Reducing high-frequency noise in slow signals	Arduino data logger (filtering temperature, humidity signals)
Robotics	Ensuring stable readings from distance sensors	Arduino robot with ultrasonic sensors (filtering echo signals)
Battery-Powered Systems	Protecting sensitive circuits from power spikes	Arduino IoT device (adding LC filter at battery input)
Audio Recording Projects	Improving recording quality by removing hum	Arduino audio recorder (filtering 50/60 Hz mains hum)

- **Band-pass filters** allow a specific range of frequencies to pass while attenuating frequencies outside this range.
- **Band-stop filters** attenuate a specific range of frequencies while allowing others to pass.

3.3 Components in first- and second-order filters

The fundamental components used to build analog filters are familiar to you from the "Introduction to Electronics" course:

Component	**Purpose in Filters**
Resistors (R)	Control current and voltage division
Capacitors (C)	Store and release electrical energy based on frequency
Inductors (L)	Oppose changes in current, depending on frequency
Operational Amplifiers (Op-Amps)	Provide gain and buffering in active filters

First-order filters typically use one reactive component (either a capacitor or an inductor) and one resistor. These simple circuits introduce a frequency-dependent behaviour that creates a gradual transition between passband and stopband.

Second-order filters introduce a second reactive component. This addition allows for steeper transitions and more selective frequency control. For example, a second-order low-pass filter might use two capacitors and two resistors, or one inductor and one capacitor combined with resistors.

Later in the book, you will see examples of how specific combinations of these components create different types of first-order filters.

3.4 Knowledge prerequisites

If you completed my course "Introduction to Electronics", you already have a strong foundation for understanding how filters work:

- You know how resistors, capacitors, and inductors behave individually in a circuit.
- You are familiar with concepts such as voltage division, time constants, and impedance.

- You have seen how capacitors block DC signals and pass AC signals, while inductors do the opposite.
- You have used Ohm's Law, Kirchhoff's Laws, and basic circuit analysis techniques to solve practical problems.

First-order filters naturally extend these ideas. For instance:

- A simple RC low-pass filter is just a resistor and a capacitor arranged to favour low-frequency signals.
- A RL high-pass filter is a resistor and an inductor arranged to favour high-frequency signals.

Second-order filters build upon this even further, requiring you to combine what you know about how capacitors and inductors behave when they interact together in more complex networks.

Throughout this book, you will see how familiar principles are applied in new ways to design and analyse filters, and you will acquire the additional mathematical tools needed to work with more advanced concepts.

3.5 Why focus on first-order filters?

This book focuses specifically on first-order analog filters because they are the most accessible and versatile type of filter:

Feature	First-Order Filter	Second-Order Filter
Roll-off rate	20 dB per decade	40 dB per decade
Circuit complexity	Simple (1 reactive component)	More complex (2 reactive components)
Frequency response	Gradual transition	Sharper transition
Applications	Basic filtering tasks	Precise filtering tasks

First-order filters are easy to design, analyse, and implement. They provide a solid entry point into the world of signal filtering without requiring complex mathematics from the start.

As you move through this book, you will learn the necessary mathematics and design techniques needed to create your own first-order filters confidently. The step-by-step approach ensures that every new concept builds naturally on what you already know.

4 First-order filters in real life applications

First-order and second-order filters are far more than academic exercises. They are essential tools in modern electronics, shaping signals, protecting circuits, and enabling communication, control, and precision. By mastering first-order filters, you gain practical skills that are directly useful across a wide range of real-world applications. The concepts you are about to learn will not only help you in designing circuits, but also deepen your understanding of how everyday technologies around you operate.

First-order filters, despite their simplicity, are used everywhere because of their efficiency, low cost, and reliability. In many cases, a first-order filter is the best solution: it performs the needed function with minimum components, low power consumption, and minimal design complexity.

Here are some key reasons why learning first-order filters is well worth your effort:

- **Real-world relevance**: Many practical circuits use first-order filters as-is, without modification.
- **Foundational knowledge**: Understanding first-order behaviour is essential even when designing more complex, higher-order systems.
- **Efficient solutions**: When size, power, and cost matter (which they almost always do), first-order filters are often preferred.

Second-order filters certainly have important applications, particularly when more precise or steeper filtering is needed. However, they build on the same ideas you are about to master in studying first-order filters.

Real-world applications of first- and second-order filters

Here's some everyday applications for these filters:

Audio applications:
When you adjust the bass or treble controls on a music player, simple first-order filters are shaping

Application Area	Typical Filter Used	Description
Audio Tone Control Systems	First-Order	RC low-pass and high-pass filters shape bass and treble tones.
Wireless Transmitter Front Ends	First- and Second-Order	Filters remove unwanted frequencies before transmission.
Sensor Signal Conditioning	First-Order	Smoothing noisy signals from analog temperature, pressure, or light sensors.
Power Supply Smoothing	First-Order	Removing ripple from rectified DC voltages.
Motor Drive Systems	First-Order	Filtering PWM signals to control motor speed smoothly.
Communication Receivers	Second-Order	Narrow band-pass filtering to isolate a communication channel.
Medical Instrumentation	First- and Second-Order	Cleaning ECG and EEG signals to remove noise without distorting the data.
Embedded Systems (e.g., Arduino)	First-Order	Filtering inputs from potentiometers, photodiodes, and other analog devices.
Robotics and Automation	First-Order	Filtering distance sensor readings for better navigation decisions.
Data Acquisition Systems	First-Order	Anti-aliasing filters before analog-to-digital conversion.

the sound. Capacitors and resistors modify the balance of low and high frequencies to match your preference.

Sensor inputs:
An Arduino reading a temperature sensor can receive noisy signals due to environmental interference. A small RC low-pass filter, using just a resistor and a capacitor, can significantly improve the quality of the sensor reading.

Motor control:
When controlling a motor's speed with PWM (Pulse Width Modulation), a first-order low-pass filter can smooth the voltage and prevent the motor from producing audible whining noises.

Wireless systems:

Before transmitting or receiving wireless signals, circuits often include filters to ensure that only the desired frequencies are used. First-order filters can remove simple spurious signals, while second-order filters are sometimes used for tighter frequency control.

4.1 What about second-order filters?

While first-order filters are sufficient for many tasks, second-order filters are valuable when:

- A sharper transition between passed and blocked frequencies is required.
- Higher selectivity is needed, such as isolating a very narrow frequency band in communication systems.
- Improved stability and control over gain and phase are critical, such as in feedback control systems.

Learning second-order filters becomes natural and much easier once you are confident with the concepts and behaviours of first-order filters.

5 The four filters: low-pass, high-pass, band-pass, band-stop

In the study of electronic filters, four fundamental types of filters are commonly encountered. Each type plays a specific role in controlling how signals of different frequencies are treated by a circuit. Understanding these filters will give you the ability to design circuits that emphasise or suppress signals based on their frequency content.

In this chapter, we will look closely at each of these four filter types, explain what they do, and discuss how they can be realised using first-order or second-order circuits.

5.1 Low-Pass Filters

A low-pass filter allows signals with frequencies below a certain cutoff frequency to pass through with little attenuation, while it attenuates signals with frequencies above the cutoff.

In other words, low-pass filters "pass the lows" and "block the highs."

Everyday Example:
In an audio system, a low-pass filter can be used to send only low-frequency signals to a subwoofer, removing unwanted higher-frequency sounds.

First-Order Implementation:
A first-order low-pass filter can be built using a resistor and a capacitor (RC circuit) or a resistor and an inductor (RL circuit). The roll-off beyond the cutoff frequency is gentle, at a rate of 20 dB per decade.

Second-Order Implementation:
A second-order low-pass filter uses two reactive components (e.g., two capacitors, or one capacitor and one inductor). The roll-off is steeper, at 40 dB per decade, providing better separation between passed and blocked frequencies.

5.2 High-Pass Filters

A high-pass filter does the opposite of a low-pass filter. It allows signals with frequencies above a certain cutoff frequency to pass through, while it attenuates signals with frequencies below the cutoff.

Everyday Example:
In audio systems, a high-pass filter can be used to remove deep, low-frequency rumble from a recording, preserving only the midrange and treble sounds.

First-Order Implementation:
A first-order high-pass filter can be constructed using a capacitor and a resistor (RC circuit) or an inductor and a resistor (RL circuit), but arranged differently from the low-pass configuration. The filter begins attenuating signals below the cutoff frequency at a rate of 20 dB per decade.

Second-Order Implementation:
A second-order high-pass filter uses two reactive components and achieves a steeper attenuation of 40 dB per decade.

5.3 Band-Pass Filters

A band-pass filter allows signals within a specific range of frequencies (a "band") to pass through while attenuating signals at frequencies lower or higher than this band. Band-pass filters are crucial when only a specific range of frequencies is needed.

Everyday Example:
In radio receivers, a band-pass filter selects the desired broadcast frequency while rejecting others.

First-Order Implementation:
A first-order band-pass filter can be made by cascading a first-order low-pass filter with a first-order high-pass filter. However, the frequency selection is not very sharp. The filter's bandwidth is relatively wide, and its ability to isolate a narrow frequency band is limited.

Second-Order Implementation:
A second-order band-pass filter offers much sharper selection, isolating a narrow band of frequencies with a steeper roll-off on both sides of the band. This is necessary when precise frequency targeting is required, such as in communication systems.

5.4 Band-Stop Filters

A band-stop filter (also called a notch filter) does the reverse of a band-pass filter. It attenuates signals within a certain range of frequencies while allowing signals outside this range to pass through.

Everyday Example:
In audio systems, a band-stop filter can be used to eliminate a specific unwanted frequency, such as the 50 Hz or 60 Hz hum from electrical mains interference.

First-Order Implementation:
A first-order band-stop filter can be made by combining a low-pass filter and a high-pass filter in parallel. Like the first-order band-pass filter, it offers only gentle attenuation and a wide transition band.

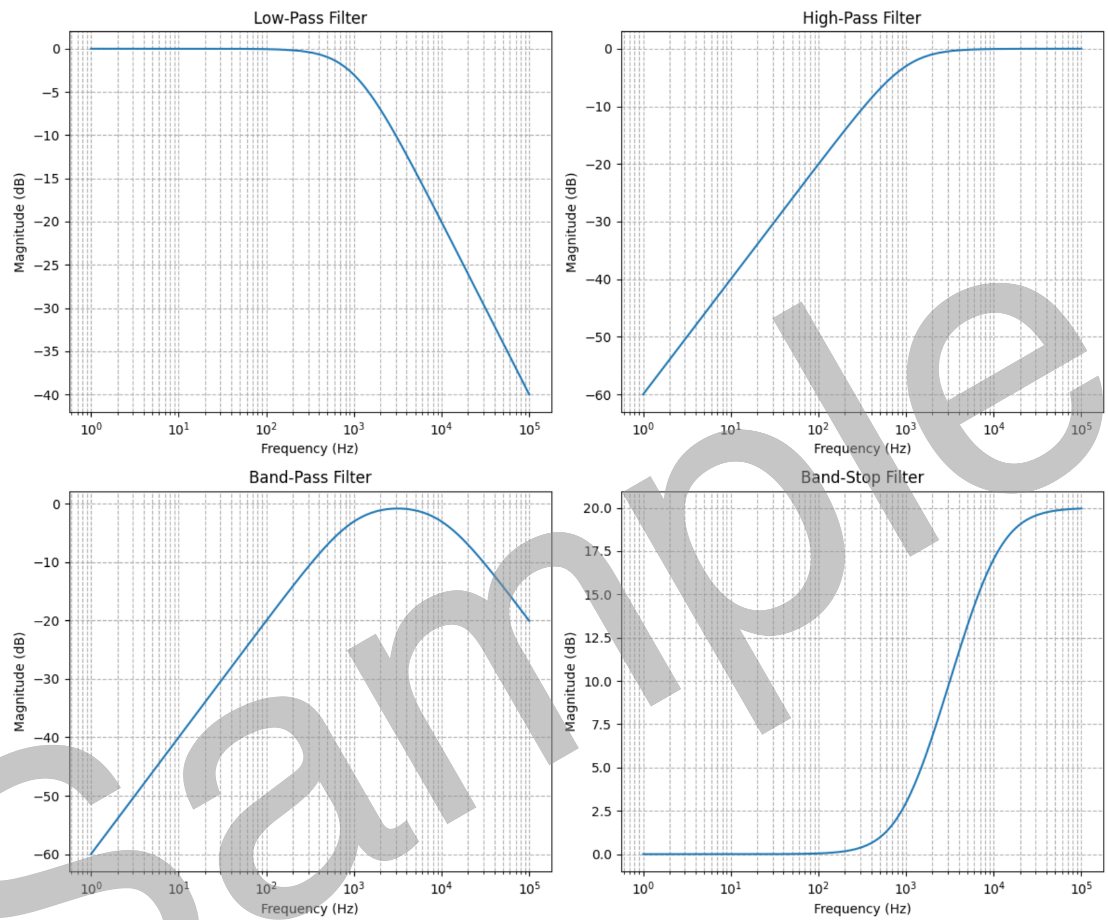
Second-Order Implementation:
A second-order band-stop filter provides a much sharper notch, attenuating a narrow range of frequencies much more effectively. This is ideal when targeting a specific, problematic frequency for removal.

5.5 Which filters can be first-order?

These plots depict the frequency response for the four types of filters:

And, this table summarises which of these filters can be implemented with a first or second order circuit:

- **Low-pass and high-pass filters** are very effective even as first-order filters, especially when gentle filtering is acceptable or desired.



Filter Type	First-Order Implementation?	Second-Order Advantage?
Low-Pass	Yes	Steeper roll-off
High-Pass	Yes	Steeper roll-off
Band-Pass	Yes, but with wide bandwidth	Sharper frequency selection
Band-Stop	Yes, but with wide stop band	Sharper, narrower notch

- **Band-pass and band-stop filters** can be implemented using first-order techniques but perform much better when built as second-order circuits, especially when narrow frequency control is important.

5.6 Python script for the plots

If you'd like to experiment with the plots, feel free to play with this Python script:

```
import numpy as np
import matplotlib.pyplot as plt

def plot_filters():
    # Frequency range (logarithmic)
    freq = np.logspace(0, 5, 500)  # 1 Hz to 100 kHz

    # Define cutoff frequencies
    f_low_cutoff = 1000  # 1 kHz
    f_high_cutoff = 10000  # 10 kHz

    # Angular frequencies
    omega = 2 * np.pi * freq
    omega_low = 2 * np.pi * f_low_cutoff
    omega_high = 2 * np.pi * f_high_cutoff

    # First-order transfer functions (magnitude)
    low_pass = 1 / np.sqrt(1 + (freq / f_low_cutoff)**2)
    high_pass = (freq / f_low_cutoff) / \
                np.sqrt(1 + (freq / f_low_cutoff)**2)

    # Band-pass and Band-stop approximations
    band_pass = (freq / f_low_cutoff) / \
                np.sqrt((1 + (freq / f_low_cutoff)**2) *
                        (1 + (freq / f_high_cutoff)**2))
    band_stop = np.sqrt((1 + (freq / f_low_cutoff)**2) /
                        (1 + (freq / f_high_cutoff)**2))

    # Create plots
    fig, axs = plt.subplots(2, 2, figsize=(12, 10))
```

```
    axs[0, 0].semilogx(freq, 20*np.log10(low_pass))
    axs[0, 0].set_title('Low-Pass Filter')
    axs[0, 0].set_xlabel('Frequency (Hz)')
    axs[0, 0].set_ylabel('Magnitude (dB)')
    axs[0, 0].grid(True, which='both', ls='--')

    axs[0, 1].semilogx(freq, 20*np.log10(high_pass))
    axs[0, 1].set_title('High-Pass Filter')
    axs[0, 1].set_xlabel('Frequency (Hz)')
    axs[0, 1].set_ylabel('Magnitude (dB)')
    axs[0, 1].grid(True, which='both', ls='--')

    axs[1, 0].semilogx(freq, 20*np.log10(band_pass))
    axs[1, 0].set_title('Band-Pass Filter')
    axs[1, 0].set_xlabel('Frequency (Hz)')
    axs[1, 0].set_ylabel('Magnitude (dB)')
    axs[1, 0].grid(True, which='both', ls='--')

    axs[1, 1].semilogx(freq, 20*np.log10(band_stop))
    axs[1, 1].set_title('Band-Stop Filter')
    axs[1, 1].set_xlabel('Frequency (Hz)')
    axs[1, 1].set_ylabel('Magnitude (dB)')
    axs[1, 1].grid(True, which='both', ls='--')

    plt.tight_layout()
    plt.show()

if __name__ == "__main__":
    plot_filters()
```

6 Example: Compare a filtered and unfiltered signal

To better understand the purpose of filters, it is very helpful to visualise the difference between an unfiltered signal and its filtered counterpart. In this chapter, we will look at a simple example using a low-pass filter. We will see, in a qualitative way, how a filter can clean up a signal by removing unwanted high-frequency noise.

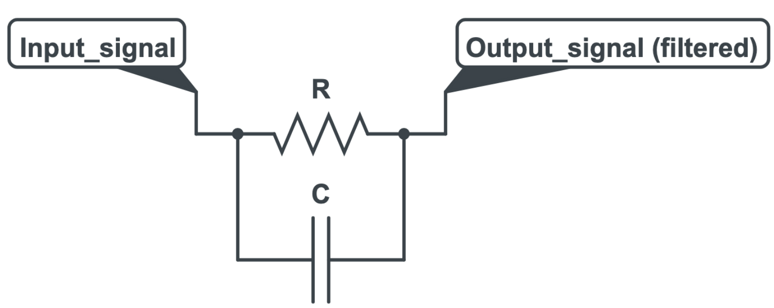
We will not dive into the mathematical details yet. The goal here is to build an intuitive understanding of the effect that a filter has on a signal. You will learn the technical reasons behind this behaviour later in the book.

6.1 Example: Low-pass filter

Suppose we have a signal that consists of two components:

- A **slow, low-frequency component** (for example, a temperature sensor reading).
- A **fast, high-frequency noise component** (for example, electrical interference from nearby equipment).

In real circuits, this kind of situation is very common. Sensors often pick up unwanted noise from their environment. We can use a low-pass filter to allow the slow signal to pass through while reducing or removing the fast noise. Here is the basic structure of the first-order RC low-pass filter we are using:

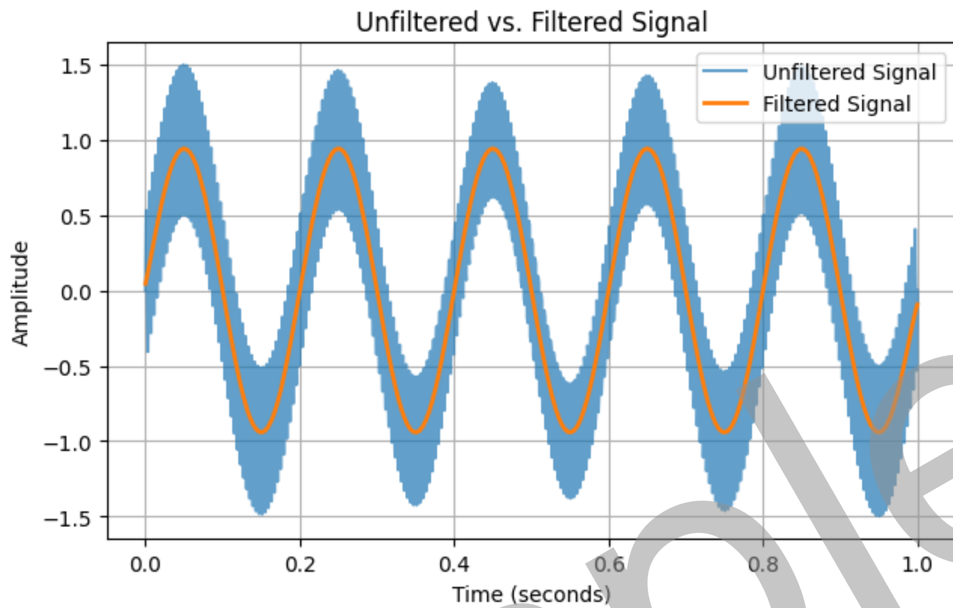


Below is a simple Python script that generates an unfiltered signal and its filtered version using a basic low-pass filter model:

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.signal import butter, filtfilt

# Create a time array
t = np.linspace(0, 1, 1000)  # 1 second duration, 1000 samples

# Create a signal: low-frequency signal + high-frequency noise
low_freq_signal = np.sin(2 * np.pi * 5 * t)  # 5 Hz component
high_freq_noise = 0.5 * np.sin(2 * np.pi * 250 * t)  # 250 Hz noise
signal = low_freq_signal + high_freq_noise

# Design a simple first-order Butterworth low-pass filter
cutoff_frequency = 20  # 20 Hz cutoff
b, a = butter(N=1,
              Wn=cutoff_frequency / (0.5 * 1000),
              btype='low')  # N=1 for first-order

# Apply the filter
filtered_signal = filtfilt(b, a, signal)

# Plotting
plt.figure(figsize=(12, 6))
plt.plot(t, signal, label='Unfiltered Signal', alpha=0.7)
plt.plot(t, filtered_signal, label='Filtered Signal', linewidth=2)
plt.xlabel('Time (seconds)')
plt.ylabel('Amplitude')
plt.title('Unfiltered vs. Filtered Signal')
plt.legend()
plt.grid(True)
plt.show()
```

This script generates a signal that combines a low-frequency sine wave with high-frequency noise. It then applies a simple first-order low-pass filter with a cutoff frequency of 20 Hz. Finally, it plots both the original and the filtered signal for easy comparison.

Here is the plot:



6.2 What do we see in the plot?

When you run this simulation, you will notice the following:

- The unfiltered signal (blue) looks very noisy and messy. It is difficult to clearly see the underlying slow movement of the signal.
- The filtered signal (orange) appears much smoother. The unwanted fast variations are greatly reduced, making it easier to see the true behaviour of the original low-frequency signal.

This is the essence of what a filter does: it improves the quality of a signal by removing unwanted parts without disturbing the useful information too much.

Later in the book, you will learn how the resistor and capacitor work together to create the filtering effect. You will also learn how to calculate the cutoff frequency and how different filters affect different types of signals. For now, it is enough to see that even a very simple circuit can have a big impact on the quality of the information that a system processes.

Part III

Dive into first order RC and RL filters

7 Introduction

First-order RC and RL filters form the backbone of many electronic systems, enabling the control and manipulation of signal frequencies in a simple yet powerful way. Filters determine how circuits respond to varying frequencies. The behaviour of these filters depends on fundamental components such as resistors, capacitors, and inductors.

In real-world applications, the signal conditioning provided by filters is critical. It ensures that only the desired frequencies are processed, thereby improving the overall performance and reliability of electronic systems. As you progress through the chapters in this section, you will learn how these components interact in simple networks to achieve precise filtering outcomes.

Understanding the principles behind first-order filters also offers insight into more complex systems. The concepts of voltage division, cutoff frequency, and phase shift that are covered here recur in multi-stage filter designs and other areas of signal processing. This knowledge serves as a stepping stone to designing circuits that address a wide range of technical challenges in electronics.

Each chapter in this section builds on the core principles of first-order filtering. The content is structured to provide a systematic exploration of both RC and RL filters, focusing on practical behaviour and theoretical underpinnings.

7.1 What is a first-order filter?

This chapter introduces the essential concept of a first-order filter—a circuit containing one reactive component paired with a resistor. It covers the mathematical foundation, showing how the first-order differential equation describes the circuit's response. This equation describes how changes in resistance, capacitance, or inductance affect the filter's performance. The chapter lays the groundwork for grasping more detailed behaviours in subsequent lessons.

7.2 RC low-pass filter behaviour

Here, the focus is on the RC low-pass filter. The chapter explains how a resistor and capacitor work together to allow low-frequency signals to pass while attenuating higher frequencies. You will explore how the time constant, defined by the product of resistance and capacitance, influences the filter's response. Practical examples illustrate the application of RC low-pass filters in reducing high-frequency noise, which is vital in audio circuits and sensor signal conditioning.

7.3 RC high-pass filter behaviour

This chapter reverses the roles of the components compared to the low-pass filter. In an RC high-pass filter, the capacitor blocks low-frequency signals and allows higher frequencies to pass. The discussion details the circuit configuration and the impact of component values on the frequency response. We'll use examples to explain how RC high-pass filters are used in situations where it is important to eliminate unwanted low-frequency interference, such as in AC coupling applications.

7.4 RL low-pass filter behaviour

Transitioning to RL filters, this chapter examines the RL low-pass filter. Unlike the RC filter, here an inductor plays a central role in allowing low frequencies while impeding high frequencies due to its natural opposition to rapid current changes. The chapter breaks down how inductive reactance interacts with resistance to create a filtering effect. This insight is particularly useful in power electronics and radio frequency circuits, where managing inductive properties is critical for circuit stability.

7.5 RL high-pass filter behaviour

In contrast to the low-pass configuration, the RL high-pass filter chapter discusses how an inductor, when configured appropriately, allows high frequencies to pass and reduces low-frequency signals. Through the examples in this chapter you will understand how the reactive properties of the inductor contribute to the overall frequency response.

7.6 Voltage division in AC using reactance

Building on basic voltage division principles from resistive circuits, this chapter introduces the concept of reactance. Here, the focus is on how voltage is divided in AC circuits when reactive components like capacitors and inductors are involved. The frequency-dependent nature of reactance is explained, showing how voltage division changes with varying signal frequencies. This concept is critical for understanding how first-order filters determine the proportion of the signal present across different circuit elements.

7.7 Cutoff frequency

The cutoff frequency is a key parameter that defines the operational limits of a filter. In this chapter, you will learn how to calculate the cutoff frequency based on the values of resistors and reactive components. You will also learn how the cutoff frequency marks the transition between the passband and the attenuation band.

7.8 Phase shift introduction

The final chapter in this section addresses the often-overlooked aspect of phase shift. First-order filters not only affect the amplitude of signals but also their timing. This chapter introduces the concept of phase shift, detailing how it arises from the interaction between resistive and reactive components. Understanding phase shift is essential for applications where signal timing is critical, such as in communication systems and digital signal processing. By grasping phase shift, you gain the tools to predict and control the temporal behaviour of signals in filtered circuits.

7.9 Practical activities, an overview

These activities reinforce theoretical concepts by guiding you through practical applications.

Activity 1: Calculating filter response

Focus on computing the filter's response at low, cutoff, and high frequencies. This exercise solidifies your understanding of frequency response and the significance of cutoff frequency in defining filter performance.

Activity 2: Simulating Filter Response

Engage with simulation tools to observe how filters behave dynamically. This hands-on practice provides insights into transient and steady-state behaviours, bridging the gap between theory and real-world circuit performance.

Activity 3: Plotting Output vs. Frequency with Python

Use Python to plot output against frequency for various RC and RL configurations. This activity encourages experimentation with different component values, enabling you to visualise the impact of changes on filter behaviour and enhancing your data analysis skills.

8 What is a first-order filter?

A first-order filter is one of the simplest types of electronic filters. It uses a single reactive component—a capacitor in RC filters or an inductor in RL filters—paired with a resistor to shape the frequency content of an input signal. The term "first-order" comes from the fact that the mathematical description of these filters involves a first-order differential equation. This simplicity leads to a gradual transition between the passband and the stopband, typically with a slope of 20 dB per decade.

> **Will you need to learn differential equations to understand first-order filters?**
> No! While first-order filters are derived from linear differential equations, a student can understand their behaviour and practical use without needing to solve such equations. You will need to know basic algebra, understand the concept of frequency, phasors (more about this later in this book) and basic circuit laws, like Ohm's and Kirchhoff's Laws.

8.1 Key characteristics

The main attributes of a first-order filter include:

- **Single Energy Storage Element:** Only one capacitor or one inductor is used.
- **Time Constant:** The response is defined by the time constant. In RC filters, the time constant is given by $\tau = RC$, while in RL filters, it is $\tau = \frac{L}{R}$.
- **Cutoff Frequency:** The cutoff frequency is the frequency at which the output signal falls to 70.7% (or -3 dB) of the input. For RC filters, it is calculated as $f_c = \frac{1}{2\pi RC}$; for RL filters, it is $f_c = \frac{R}{2\pi L}$.
- **Gentle Roll-Off:** With a slope of $\pm$20 dB per decade, the filter gradually attenuates frequencies beyond the cutoff, unlike the steeper slopes seen in higher-order designs.

8.2 First-Order vs. Higher-Order Filters

First-order filters use a single reactive element, resulting in a smooth transition between passed and rejected frequencies. In contrast, second-order or higher-order filters incorporate two or more reactive components, which leads to:

- **Steeper Roll-Off:** Second-order filters typically offer a roll-off rate of 40 dB per decade, providing a sharper distinction between the passband and stopband.

- **Complex Response Characteristics:** Higher-order filters can exhibit resonant peaks and more pronounced phase shifts, which can be advantageous in applications requiring precise filtering but add complexity to the design.

What is "Roll-Off"?
Roll-off is the rate at which a filter attenuates the signal amplitude beyond its cutoff frequency, typically measured in decibels per decade or per octave.

8.3 Summary Table of First-Order Filters

The following table summarises the main attributes of common first-order filters:

Filter	**Contains**	$H(s)$	f_c	**Roll–Off**	**Phase Shift**
RC Low-Pass	R, C	$H(s) = \frac{1}{1+sRC}$	$f_c = \frac{1}{2\pi RC}$	-20 dB/decade	$0°$ to $-90°$
RC High-Pass	R, C	$H(s) = \frac{sRC}{1+sRC}$	$f_c = \frac{1}{2\pi RC}$	20 dB/decade	$0°$ to $+90°$
RL Low-Pass	R, L	$H(s) = \frac{R}{R+sL}$	$f_c = \frac{R}{2\pi L}$	-20 dB/decade	$0°$ to $-90°$
RL High-Pass	R, L	$H(s) = \frac{sL}{R+sL}$	$f_c = \frac{R}{2\pi L}$	20 dB/decade	$0°$ to $+90°$

Note: In these expressions, s represents the complex frequency (with $s = j\omega$, where ω is the angular frequency in radians per second).

8.4 Adjusting the cutoff frequency

Imagine turning a knob that gradually adjusts how much of a signal is allowed to pass through a circuit. In a first-order filter, this adjustment is controlled by the resistor and the reactive component working together. At frequencies well within the passband, most of the signal passes through unchanged. As the frequency approaches the cutoff value, the filter starts to attenuate the signal gradually. Beyond this point, the filter increasingly reduces the signal's amplitude.

For example, an RC low-pass filter acts like a barrier to high frequencies, whereas an RC high-pass filter behaves like a gate that favours higher frequencies and reduces lower ones. RL filters operate on similar principles, but the reactive element is an inductor, which naturally opposes changes in current.

9 RC low-pass filter bevaviour

An RC low-pass filter is designed to allow low-frequency signals to pass through with little attenuation while gradually reducing the amplitude of higher frequency signals. In this configuration, a resistor and a capacitor work together so that at low frequencies the capacitor presents a high impedance, resulting in the output voltage closely following the input voltage. As the frequency increases, the impedance of the capacitor decreases, which causes more of the high-frequency signal to be shunted to ground. This behaviour creates a smooth transition between the passband and the stopband, with the cutoff frequency marking the point where the output drops to about 70.7% (or -3 dB) of the input signal.

> **What is "passband"?**
> The range of frequencies that a filter allows to *pass* through with minimal attenuation. These frequencies remain largely unchanged by the filter.

> **What is "stopband"?**
> The range of frequencies that a filter significantly *attenuates* or *blocks*, reducing their amplitude substantially.

9.1 Circuit Diagram

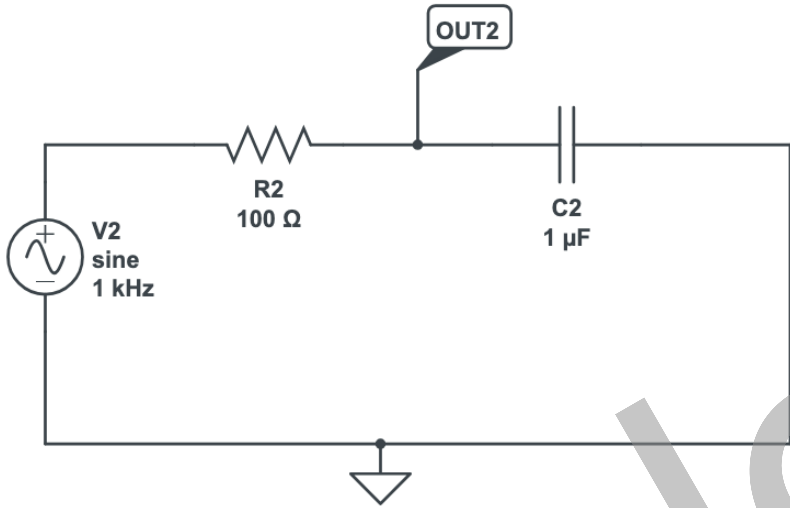
The following schematic represents a basic RC low-pass filter:

A basic RC low-pass filter

In this circuit, the resistor (R) is connected in series with the input voltage, while the capacitor (C) is connected between the node after the resistor and ground. This arrangement forms a voltage divider where the capacitor's impedance decreases with increasing frequency, leading to signal attenuation at higher frequencies. In this example, the input voltage source is an alternating sine wave.

9.2 Mathematical underpinning

If you are curious about the underlying mathematics that describe the behaviour of the RC low-pass filter, continue to read this segment. Otherwise, you will find the most important take-away



formula (the cut-off frequency) at the end of the segment. The behaviour of the RC low-pass filter is characterised by its transfer function. The transfer function is given by:

$$H(s) = \frac{1}{1 + sRC}$$

To analyse the filter's response to sinusoidal inputs, we substitute the complex frequency s with $j\omega$, where Ω is the angular frequency in radians per second.

What is a "complex frequency"?
The complex frequency s is a variable used in the Laplace transform, defined as $s = \sigma + j\omega$. It combines an exponential decay or growth component (σ) with an oscillatory component (ω), making it essential for analysing the behaviour of linear time-invariant systems. Thankfully, as a new student of first-order filters, you don't need to learn the concept of complex frequency s to be able to understand how these filters work, and to be to use them.

What is the Laplace Transform? The Laplace transform is a mathematical operation that converts a time-domain function, $f(t)$, into a complex frequency-domain function, $F(s)$, where $s = \sigma + j\omega$. This transformation simplifies the analysis of linear time-invariant circuits by turning differential equations into algebraic ones. For circuits such as first-order filters, the Laplace transform allows us to derive transfer functions, analyse stability, and predict transient and steady-state behaviour efficiently.

This substitution results in:

$$H(j\omega) = \frac{1}{1 + j\omega RC}$$

The magnitude of the transfer function, which represents the filter's frequency response, is determined by:

$$|H(j\omega)| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

At low frequencies, where $\omega RC \ll 1$, the magnitude approaches 1, meaning the output is nearly equal to the input. As the frequency increases, the term $(\omega RC)^2$ becomes significant, and the magnitude decreases accordingly. The cutoff frequency, defined as the frequency at which the output drops to $\frac{1}{\sqrt{2}}$ of the input (approximately -3 dB), is given by:

$$f_c = \frac{1}{2\pi RC}$$

This mathematical model is derived using the voltage division principle, where the capacitor's impedance is expressed as $\frac{1}{j\omega C}$, making the filter's response inherently dependent on frequency.

9.3 Frequency Response Plot

Below is a Python code snippet that generates a plot of the filter's frequency response. The plot illustrates the signal attenuation (in decibels) as a function of frequency, highlighting the gradual roll-off characteristic of the RC low-pass filter.

```
import numpy as np
import matplotlib.pyplot as plt

# Define resistor and capacitor values
R = 1e3  # Resistance in ohms
C = 1e-6  # Capacitance in farads

# Calculate the cutoff frequency
fc = 1 / (2 * np.pi * R * C)
```

```
# Define frequency range for the plot (10 Hz to 1 MHz)
frequencies = np.logspace(1, 6, num=500)
omega = 2 * np.pi * frequencies

# Calculate the magnitude of the transfer function for the low-pass filter
H_mag = 1 / np.sqrt(1 + (omega * R * C)**2)
H_db = 20 * np.log10(H_mag)

# Create the plot
plt.figure(figsize=(8, 4))
plt.semilogx(frequencies, H_db)
plt.xlabel("Frequency (Hz)")
plt.ylabel("Magnitude (dB)")
plt.title("Frequency Response of RC Low-Pass Filter")
plt.grid(True, which="both", linestyle="--")

# Mark the cutoff frequency with a red dashed line
plt.axvline(x=fc,
            color="red",
            linestyle="--",
            label=f"Cutoff Frequency: {fc:.1f} Hz")
plt.legend()

plt.show()
```

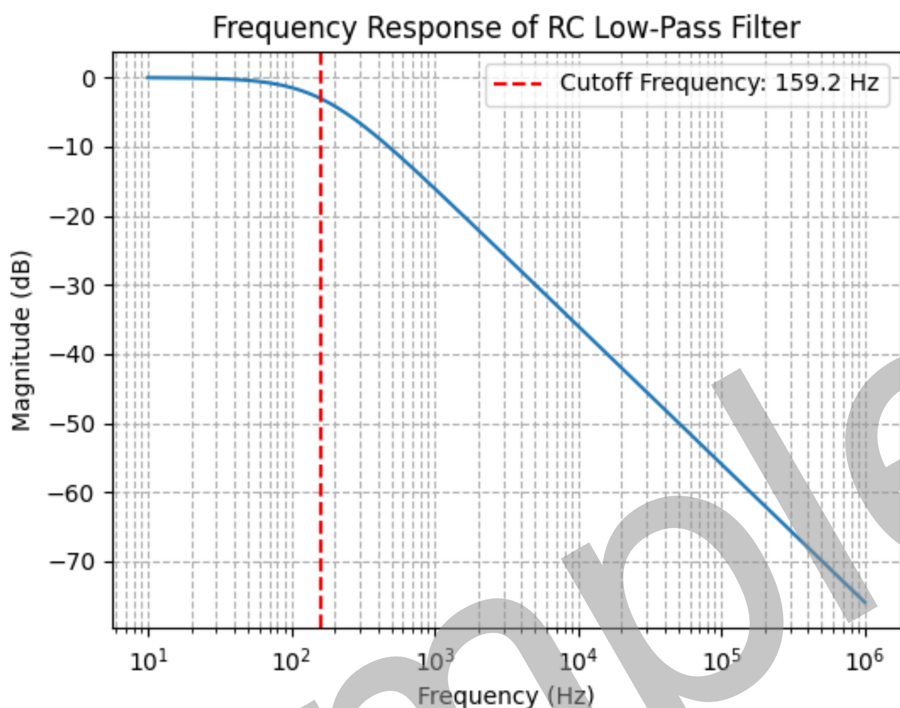
When executed, this code produces a plot showing that the filter maintains a flat response at low frequencies and begins to attenuate the signal progressively above the cutoff frequency.

The frequency response of a RC low-pass filter

The red vertical line marks the cutoff frequency, which is also calculated by the script. Using this code, you can experiment with various values for R and C to see how they effect the frequency response of the filter.

9.4 Key takeaways

In summary, the RC low-pass filter is a foundational circuit that illustrates several core concepts in signal processing:



- The filter consists of a resistor in series and a capacitor to ground, forming a voltage divider whose behaviour changes with frequency.

- At low frequencies, the capacitor acts as an open circuit, allowing most of the input signal to appear at the output. At high frequencies, its decreasing impedance diverts the signal to ground.

- The transfer function, $H(j\omega) = \frac{1}{1+j\omega RC}$, and its magnitude, $|H(j\omega)| = \frac{1}{\sqrt{1+(\omega RC)^2}}$, mathematically describe this behaviour.

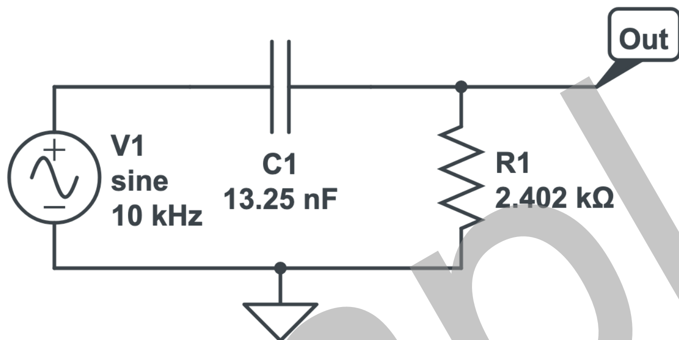
- The cutoff frequency, $f_c = \frac{1}{2\pi RC}$, defines the boundary between the passband (where signals pass with little attenuation) and the stopband (where signals are significantly reduced).

10 RC high-pass filter behaviour

An RC high-pass filter is designed to allow high-frequency signals to pass through while attenuating low-frequency signals. In this configuration, a resistor and a capacitor are arranged so that at low frequencies the capacitor behaves like an open circuit, blocking most of the signal. As the frequency increases, the capacitor's impedance decreases, allowing the high-frequency components to pass with minimal attenuation. This filter is especially useful in applications where it is necessary to eliminate low-frequency noise or DC offset from a signal.

10.1 Circuit Diagram

Below is a simple schematic representation of an RC high-pass filter:



A basic RC high-pass filter

In this circuit, the capacitor (C) is connected in series with the input voltage, and the resistor (R) is connected from the node between the capacitor and resistor to ground. This configuration forms a voltage divider in which the capacitor's impedance decreases with increasing frequency, thereby allowing high frequencies to appear at the output.

10.2 Mathematical underpinning

As with the low-pass filter in the previous chapter, I am providing you with the underlying mathematics that describe the behaviour of the RC high-pass filter here. If you are interested in these details, then continue to read this segment. Otherwise, you will find the most important take-away formula (the cut-off frequency) at the end of the segment. The behaviour of the RC high-pass filter is described by its transfer function. The transfer function for an RC high-pass filter is given by:

$$H(s) = \frac{sRC}{1 + sRC}$$

To analyse the filter's response to sinusoidal inputs, we substitute s with $j\omega$, where Ω is the angular frequency in radians per second. Remember, from the previous chapter, s is the complex frequency used in the Laplace transform. This substitution gives:

$$H(j\omega) = \frac{j\omega RC}{1 + j\omega RC}$$

The magnitude of the transfer function, which indicates the filter's frequency response, is calculated as:

$$|H(j\omega)| = \frac{\omega RC}{\sqrt{1 + (\omega RC)^2}}$$

At low frequencies (when $\omega RC \ll 1$), the magnitude approaches 0, meaning the output is greatly attenuated. Conversely, at high frequencies (when $\omega RC \gg 1$), the magnitude approaches 1, allowing the high-frequency signals to pass through. The cutoff frequency, which marks the transition between these two regions, is defined as:

$$f_c = \frac{1}{2\pi RC}$$

This mathematical framework is derived from the voltage division principle, taking into account the frequency-dependent impedance of the capacitor, expressed as $\frac{1}{j\omega C}$.

10.3 Frequency Response Plot

The following Python code snippet demonstrates how to plot the frequency response of an RC high-pass filter. The plot shows the magnitude of the output signal in decibels (dB) as a function of frequency.

```
import numpy as np
import matplotlib.pyplot as plt

# Define resistor and capacitor values
R = 1e3  # Resistance in ohms
C = 1e-6  # Capacitance in farads

# Calculate the cutoff frequency
fc = 1 / (2 * np.pi * R * C)

# Frequency range for the plot (10 Hz to 1 MHz)
frequencies = np.logspace(1, 6, num=500)
```

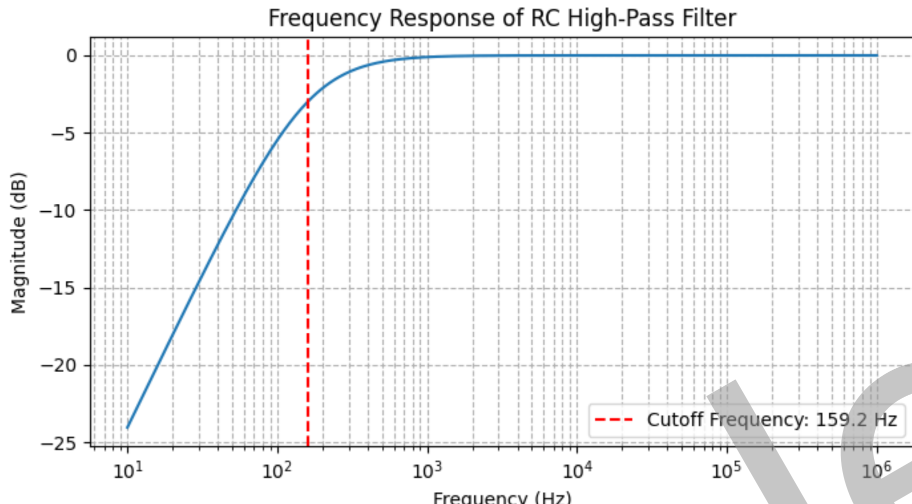
```
omega = 2 * np.pi * frequencies

# Calculate the magnitude of the transfer function for the high-pass filter
H_mag = (omega * R * C) / np.sqrt(1 + (omega * R * C)**2)
H_db = 20 * np.log10(H_mag)

# Create the plot
plt.figure(figsize=(8, 4))
plt.semilogx(frequencies, H_db)
plt.xlabel("Frequency (Hz)")
plt.ylabel("Magnitude (dB)")
plt.title("Frequency Response of RC High-Pass Filter")
plt.grid(True, which="both", linestyle="--")

# Mark the cutoff frequency
plt.axvline(x=fc,
            color="red",
            linestyle="--",
            label=f"Cutoff Frequency: {fc:.1f} Hz")
plt.legend()

plt.show()
```

Executing this code produces a plot that illustrates how the filter attenuates low frequencies while allowing high frequencies to pass, with the transition occurring around the cutoff frequency $f_c = \frac{1}{2\pi RC}$. The red vertical line marks the cutoff frequency, which is also calculated by the script.

The frequency response of a RC high-pass filter

10.4 Key Takeaways

- The RC high-pass filter uses a capacitor in series with the input and a resistor to ground to block low-frequency signals and pass high-frequency signals.
- Its transfer function is given by $H(j\omega) = \frac{j\omega RC}{1+j\omega RC}$, which shows that the filter's behaviour is frequency dependent.



- The magnitude of the transfer function, $|H(j\omega)| = \frac{\omega RC}{\sqrt{1+(\omega RC)^2}}$, confirms that the output is nearly zero at low frequencies and approaches unity at high frequencies.
- The cutoff frequency, $f_c = \frac{1}{2\pi RC}$, serves as the boundary between the stopband and the passband.

Part IV

A primer of algebra for first-order filter analysis

11 Formula map

Here is a formula map that shows the relationships between RC, ω_c, ω, and how substitution works in transfer functions:

1. **Cutoff angular frequency** (ω_c) definition:

$$\omega_c = \frac{1}{RC}$$

2. **Rearranging for RC**:

$$RC = \frac{1}{\omega_c}$$

3. **Transfer function before substitution** (example: low-pass filter):

$$H(j\omega) = \frac{1}{1 + j\omega RC}$$

4. **Substituting RC in terms of ω_c**:

$$H(j\omega) = \frac{1}{1 + j\left(\frac{\omega}{\omega_c}\right)}$$

5. **Relationship between ω and f**:

$$\omega = 2\pi f$$

$$\omega_c = 2\pi f_c$$

Thus:

$$\frac{\omega}{\omega_c} = \frac{f}{f_c}$$

Here is the final Simplified Transfer Functions (in terms of frequency ratio):

- **Low-pass filter**:

$$H(f) = \frac{1}{1 + j\left(\frac{f}{f_c}\right)}$$

- **High-pass filter**:

$$H(f) = \frac{j\left(\frac{f}{f_c}\right)}{1 + j\left(\frac{f}{f_c}\right)}$$

Try to remember these points:

- **Substituting** RC **with** $1/\omega_c$ simplifies expressions.
- **The ratio** f/f_c **or** ω/ω_c always appears in transfer functions.
- **This ratio controls the behaviour** of the filter across different frequencies.